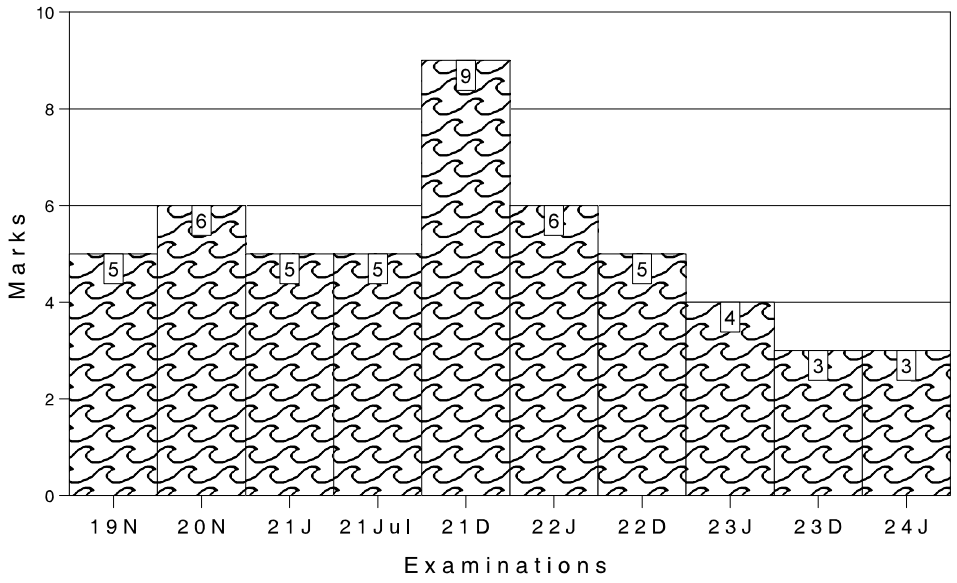
CHAPTER	
1	# RATIO AND PROPORTION, INDICES, LOGARITHMS

Marks of Objective, Short Notes, Distinguish Between, Descriptive & Practical Questions

Legend

Objective Short Notes Distinguish Descriptive Practical



MULTIPLE CHOICE QUESTIONS AND ANSWERS

2009 - JUNE

[1] If $\frac{p}{q} = -\frac{2}{3}$ then the value of $\frac{2p+q}{2p-q}$ is:

(a) 1 (b) -1/7

(c) 1/7 (d) 7 (1 mark)

Answer:

(c) $\frac{p}{q} = \frac{-2}{3}$

So, $P = \frac{-2q}{3}$ .....(i)

Now, $\frac{2p+q}{2p-q}$

Substituting the value of p from (i)

$$= \frac{2\left(\frac{-2q}{3}\right)+q}{2\left(\frac{-2q}{3}\right)-q}$$

$$= \frac{\frac{-4q}{3}+q}{\frac{-4q}{3}-q}$$

$$= \frac{\frac{-4q+3q}{3}}{\frac{-4q-3q}{3}}$$

$$= \frac{-q}{3} \times \frac{3}{-7q}$$

$$= \frac{1}{7}$$

[2] Fourth proportional to x, 2x, (x+1) is:

(a) (x + 2)
(b) (x – 2)
(c) (2x + 2)
(d) (2x – 2) (1 mark)

Answer:

(c) Let the fourth proportional to x, 2x, (x + 1) be t, then,

$$\frac{x}{2x} = \frac{x+1}{t}$$

$$\frac{1}{2} = \frac{x+1}{t}$$

$t = 2x + 2$

$\therefore$ Fourth proportional to x, 2x, (x + 1) is (2x + 2)

i.e. x: 2x :: (x + 1) : (2x +2)

[3] If $x = 3^{1/3} + 3^{-1/3}$ then find value of $3x^3 - 9x$

(a) 3
(b) 9
(c) 12
(d) 10 (1 mark)

Answer:

(d) $x = 3^{1/3} + 3^{-1/3}$ ..... (1)

On cubing both sides, we get

$$x^3 = (3^{1/3} + 3^{-1/3})^3$$

$$x^3\ 3 + 3^{-1} + 3 \times 3^{1/3} \times \frac{1}{3^{1/3}}(3^{1/3} + 3^{-1/3})$$

$$x^3 = 3 + \frac{1}{3} + 3\ (3^{1/3} + 3^{-1/3})$$

$$x^3 = 3 + \frac{1}{3} + 3x \quad \text{[Using (1)]}$$

$$x^3 - 3x = \frac{9+1}{3}$$

$$3(x^3 - 3x) = 10$$

$\therefore\ 3x^3 - 9x = 10$

[4] Find the value of : $[1-\{1-(1-x^2)^{-1}\}^{-1}]^{-1/2}$

(a) 1/x

(b) x

(c) 1

(d) None of these. (1 mark)

Answer:

(b) $[1-\{1-(1-x^2)^{-1}\}^{-1}]^{-½}$

$$= \left[1-\left\{1-\frac{1}{1-x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \left[1-\left\{\frac{1-x^2-1}{1-x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \left[1-\left\{\frac{-x^2}{1-x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \left[1-\left\{\frac{1-x^2}{x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \left[1+\frac{1-x^2}{x^2}\right]^{-1/2} = \left[\frac{x^2+1-x^2}{x^2}\right]^{-1/2}$$

$$= \left[\frac{1}{x^2}\right]^{-1/2} = (x^2)^{1/2}$$

$$= x$$

[5] log (m + n) = log m + log n, m can be expressed as:

(a) $m = \frac{n}{n-1}$

(b) $m = \frac{n}{n+1}$

(c) $m = \frac{n+1}{n}$

(d) $m = \frac{n+1}{n-1}$ (1 mark)

Answer:

(a) $\log (m + n) = \log m + \log n$

$\log (m + n) = \log (m\, n)$ [$\because$ $\log (ab) = \log a + \log b$]

Taking Antilog on both side

Antilog [log (m + n)] = Antilog [log mn]

$\therefore$ $m + n = mn$

$mn - m = n$

$m (n - 1) = n$

$m = \frac{n}{n - 1}$

[6] $\log_4 (x^2 + x) - \log_4(x+1) = 2$.

Find x

(a) 16

(b) 0

(c) – 1

(d) None of these. (1 mark)

Answer:

(a) $\text{Log}_4 (x^2 + x) - \text{Log}_4 (x + 1) = 2$

$\text{Log}_4 \left(\frac{x^2 + x}{x + 1}\right) = 2$[$\therefore$ $\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$]

$4^2 = \frac{x^2 + x}{x + 1}$

$16 = \frac{x^2 + x}{x + 1}$

$16x + 16 = x^2 + x$

$x^2 - 15x - 16 = 0$

$x^2 - 16x + x - 16 = 0$

$x (x - 16) + 1 (x - 16) = 0$

$(x + 1) (x - 16) = 0$

$x = -1$ or $x = 16$

Since $x = -1$ is not possible therefore $x = 16$

2009 - DECEMBER

[7] $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(a) ½
(b) 3/2
(c) 2/3
(d) 1/3 (1 mark)

Answer:

(b) $2^n + 2^{n-1}/2^{n+1} - 2^n$

$\frac{2^{n-1}(2+1)}{2^n(2-1)} = \frac{3}{2}$

[8] If $2^x \times 3^y \times 5^z = 360$ Then what is the value of x, y, z,?

(a) 3, 2, 1
(b) 1, 2, 3
(c) 2, 3, 1
(d) 1, 3, 2 (1 mark)

Answer:

(a) $2^x \times 3^y \times 5^z = 360$. .....(1)

The factors of 360 are:

$2^3 \times 3^2 \times 5$.

$\therefore\ 2^3 \times 3^2 \times 5^1 = 360$.....(2)

On comparing (1) and (2), we get;

$x = 3$, $y = 2$ and $z = 1$

[9] Find the value of $[\log_{10}\sqrt{25} - \log_{10}(2)^3 + \log_{10}(4)^2]^x$

(a) x
(b) 10
(c) 1
(d) None. (1 mark)

Answer:

(c) $[\log_{10} \sqrt{25} - \log_{10}(2^3) + \log_{10}(4^2)]^x$

$= [\log_{10} 5 - 3\log_{10} 2 + \log_{10}(2^4)]^x$

$= [\log_{10} 5 - 3\log_{10} 2 + 4\log_{10} 2]^x$

$= [\log_{10} 5 + \log_{10} 2^2]^x$

$= [\log_{10}(5 \times 2)]^x$ [$\because \log(mn) = \log m + \log n$]

$= [\log_{10} 10]^x$

$= 1^x$ [$\therefore \log_a a = 1$]

$= 1$

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[10] If $\log_a b + \log_a c = 0$ then

(a) $b = c$ (b) $b = -c$

(c) $b = c = 1$ (d) b and c are reciprocals. (1 mark)

Answer:

(d) $\log_a b + \log_a c = 0$

$\log_a bc = 0$

$a^0 = bc$

$bc = 1$

$\therefore b = \frac{1}{c}$

So, b and c are reciprocals.

[11] What must be added to each term of the ratio 49 : 68, so that it becomes 3 : 4 ?

(a) 3 (b) 5

(c) 8 (d) 9 (1 mark)

Answer:

(c) Let the number added be x

$\frac{49 + x}{68 + x} = \frac{3}{4}$

$196 + 4x = 204 + 3x$

$x = 8$

[12] The students of two classes are in the ratio 5 : 7, if 10 students left from each class, the remaining students are in the ratio of 4 : 6 then the number of students in each class is:

(a) 30, 40 (b) 25, 24

(c) 40, 60 (d) 50, 70 (1 mark)

Answer:

(d) Let the ratio be 5x : 7x

If 10 student left, Ratio became 4 : 6

$$\frac{5x - 10}{7x - 10} = \frac{4}{6}$$

$30x - 60 = 28x - 40$

$2x = 20$

$x = 10$

$\therefore$ No. of students in each class is 5x and 7x

i.e. 50, 70

2010 - DECEMBER

[13] The value of

$2 \log x + 2 \log x^2 + 2 \log x^3 + \text{-------+}$

$2 \log x^n$ will be :

(a) $\frac{n(n+1)\log x}{2}$ (b) $n(n+1) \log x$

(c) $n^2 \log x$ (d) None of these. (1 mark)

Answer:

(b) $2 \log x + 2 \log x^2 + 2\log x^3 + ...$

$2[\log x + \log x^2 + \log x^3 + ...]$

$2[\log x + 2\log x + 3\log x + ...]$

$2 \log x[1 + 2 + 3 ..... n]$

$2 \log x \times \frac{n(n+1)}{2}$

$= n(n + 1) \log x$

[14] The recurring decimal 2.7777..... can be expressed as:

(a) 24/9
(b) 22/9
(c) 26/9
(d) 25/9 (1 mark)

Answer:

(d) 2.7777

$2 + 0.7 + 0.07 + 0.007 + ...$

$2 + \left(\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + ... \right)$

$2 + 7 \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + ... \right)$

$2 + 7 \left(\frac{1/10}{1 - 1/10} \right)$

$= 2 + 7 \times \frac{1}{9}$

$= 2 + \frac{7}{9}$

$= \frac{18 + 7}{9}$

$= \frac{25}{9}$

[15] Solve : $\left(\frac{\log_{10} x - 3}{2} \right) + \left(\frac{11 - \log_{10} x}{3} \right) = 2$

(a) 10^{-1} (b) 10^{2}
(c) 10 (d) 10^{3} (1 mark)

Answer:

(a) $\left(\frac{\log_{10} x - 3}{2} \right) + \left(\frac{11 - \log_{10} x}{3} \right) = 2$

$3 \log_{10} x - 9 + 22 - 2 \log_{10} x = 12$

$\log_{10} x + 13 = 12$

$\log_{10} x = -1$

$x = 10^{-1}$

[16] If A:B = 2:5, then (10A + 3B):(5A + 2B) is equal to:

(a) 7 : 4 (b) 7 : 3

(c) 6 : 5 (d) 7 : 9 (1 mark)

Answer:

(a) $\frac{A}{B} = \frac{2}{5} = \frac{2k}{5k}$

$\frac{10A + 3B}{5A + 2B} = \frac{20k + 15k}{10k + 10k} = \frac{35k}{20k}$

$= \frac{35}{20}$

$= \frac{7}{4}$

2011 - JUNE

[17] If n = m! where ('m' is a positive integer > 2) then the value of :

$$\frac{1}{\log_2{}^n} + \frac{1}{\log_3{}^n} + \frac{1}{\log_4{}^n} + \ldots\ldots\ldots + \frac{1}{\log_m{}^n}$$

(a) 1 (b) 0

(c) -1 (d) 2 (1 mark)

Answer:

(a) Given : n = M ! for M $\geq$ 2

$$\frac{1}{\log_2{}^n} + \frac{1}{\log_3{}^n} + \frac{1}{\log_4{}^n} + \ldots\ldots\ldots\ldots + \frac{1}{\log_m{}^n}$$

or, $= \log_n{}^2 + \log_n{}^3 + \log_n{}^4 + \ldots\ldots\ldots\ldots + \log_n{}^m$ $\left(\because \log_b{}^a = \frac{1}{\log_a{}^b}\right)$

$= \log_n (2 \times 3 \times 4 \times \ldots\ldots\ldots \times m)$ $\left(\because \log^{(mn)} = \log^m + \log^n\right)$

$= \log_n (m!)$

$= \log_n{}^n$

$= 1$

[18] In a film shooting, A and B received money in a certain ratio and B and C also received the money in the same ratio. If A gets ₹ 1,60,000 and C gets ₹ 2,50,000. Find the amount received by B ?

(a) ₹ 2,00,000 (b) ₹ 2,50,000

(c) ₹ 1,00,000 (d) ₹ 1,50,000 (1 mark)

Answer:

(a) Given : A : B = B : C

$\Rightarrow$ $B^2 = A \times C$

or $B = \sqrt{A \times C}$

& A = 1,60,000 ; C = 2,50,000

$\therefore$ $B = \sqrt{1,60,000 \times 2,50,000}$

B = 2,00,000

2011 - DECEMBER

[19] The ratio Compounded of 4:5 and sub-duplicate of "a":9 is 8:15. Then Value of "a" is:

(a) 2 (b) 3

(c) 4 (d) 5 (1 mark)

Answer:

(c) Sub duplicate ratio of a : 9 = $\sqrt{a}$: $\sqrt{9}$, Compound Ratio (C.R.) = 8:15

Compound Ratio of 4 : 5 and sub duplicate ratio of a : 9 is given by

$$C.R = \frac{4}{5} \times \frac{\sqrt{a}}{\sqrt{9}}$$

$$\frac{8}{15} = \frac{4}{5} \times \frac{\sqrt{a}}{\sqrt{9}}$$

$$\sqrt{a} = \frac{8 \times 5 \times \sqrt{9}}{15 \times 4}$$

$$\sqrt{a} = \frac{8 \times 5 \times 3}{15 \times 4}$$

$$\sqrt{a} = 2$$

On squaring $(\sqrt{a})^2 = 2^2$

$$a = 4$$

[20] If $\log_2 x + \log_4 x = 6$, then the Value of x is :

(a) 16 (b) 32

(c) 64 (d) 128 (1 mark)

Answer:

(a) If $\log_2 x + \log_4 x = 6$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 4} = 6$$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 2^2} = 6$$

$$\frac{\log x}{\log 2} + \frac{\log x}{2\log 2} = 6$$

$$\frac{\log x}{\log 2}\left[1 + \frac{1}{2}\right] = 6$$

$$\frac{\log x}{\log 2} \times \frac{3}{2} = 6$$

$$\frac{\log x}{\log 2} = 6 \times \frac{2}{3}$$

$$\frac{\log x}{\log 2} = 4$$

$$\log x = 4 \log 2$$

$$\log x = \log 2^4$$

$$x = 2^4$$

$$x = 16$$

[21] If X Varies inversely as square of Y and given that Y = 2 for X = 1, then the Value of X for Y = 6 will be:

(a) 3

(b) 9

(c) 1/3

(d) 1/9 (1 mark)

Answer:

(d) Given x varies inversely as square of y

i. e. $x \alpha \frac{1}{y^2}$

$x = k \frac{1}{y^2}$

$x = \frac{k}{y^2}$.....(1)

Given x = 1, y = 2 then

$1 = \frac{k}{(2)^2} \Rightarrow k = 1 \times 4 = 4$

Now putting $y = 6_1$ k = 4 in equation (1)

$x = \frac{4}{6^2}$

$x = \frac{4}{36} = \frac{1}{9}$

2012 - JUNE

[22] The value of $\frac{(3^{n+1} + 3^n)}{(3^{n+3} - 3^{n+1})}$ is equal to:

(a) 1/5 (b) 1/6

(c) 1/4 (d) 1/9 (1 mark)

Answer:

(b) $\frac{3^{n+1} + 3^n}{3^{n+3} - 3^{n+1}} = \frac{3^n . 3^1 + 3^n}{3^n . 3^3 - 3^n . 3^1}$

$= \frac{3^n (3^1 + 1)}{3^n (3^3 - 3)}$

$= \frac{(3 + 1)}{(27 - 3)}$

$= \frac{4}{24}$

$= \frac{1}{6}$

[23] If $\log_x y = 100$ and $\log_2 x = 10$, then the value of 'y' is:

(a) 2^{10}

(b) 2^{100}

(c) $2^{1,000}$

(d) $2^{10,000}$ (1 mark)

Answer:

(c) Given $\log_x y = 100$ .....(1)

$\log_2 x = 10$.....(2)

Multiply eq (1) & (2)

$\log_x y \cdot \log_2 x = 100 \times 10$

$$\frac{\log y}{\log x} \times \frac{\log x}{\log 2} = 1{,}000$$

$\log y = 1{,}000 \log 2$

$\log y = \log 2^{1,000}$

$\Rightarrow y = 2^{1,000}$

[24] Which of the numbers are not in proportion?

(a) 6, 8, 5, 7

(b) 7, 3, 14, 6

(c) 18, 27, 12, 18

(d) 8, 6, 12, 9 (1 mark)

Answer:

(a) If say a, b, c, d are in proportion they bear a common ratio that is

$$\Rightarrow \quad \frac{a}{b} = \frac{c}{d}$$

Option (A) $\frac{6}{8} \neq \frac{5}{7}$

Option (B) $\frac{7}{3} = \frac{14}{6}$

Option (C) $\frac{18}{27} = \frac{12}{18}$

Option (D) $\frac{8}{6} = \frac{12}{9}$

2012 - DECEMBER

[25] Find the value of x, if $x\,(x)^{1/3} = (x^{1/3})^x$

(a) 3 (b) 4

(c) 2 (d) 6 (1 mark)

Answer:

(b) If $x^1\,(x)^{1/3} = (x^{1/3})^x$

$$x^{1+1/3} = x^{\frac{1}{3}x}$$

$$\Rightarrow x^{4/3} = x^{\frac{1}{3}x}$$

on comparing

$$\frac{4}{3} = \frac{x}{3}$$

$3x = 12 \Rightarrow x = 4$

[26] Which of the following is true.

If $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = \frac{1}{abc}$

(a) log (ab + bc + ca) = abc (b) $\log\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = abc$

(c) log (abc) = 0 (d) log (a + b + c) = 0 (1 mark)

Answer:

(d) Given

$$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = \frac{1}{abc}$$

$$\frac{c + a + b}{abc} = \frac{1}{abc}$$

$a + b + c = 1$

taking log on both side

$\log (a + b + c) = \log 1$

$\log (a + b + c) = 0$

[27] Find two numbers such that mean proportional between them is 18 and third proportional between them is 144

(a) 9, 36 (b) 8, 32

(c) 7, 28 (d) 6, 24 (1 mark)

Answer:

(a) Let two Nos. be x and y

Mean proportion between x and y is 18

So, x, 18, y are in proportion

x : 18 :: 18 : y

$\frac{x}{18} = \frac{18}{y}$

$xy = 324$

$x = \frac{324}{y}$ _____ (1)

If third proportion between x & y be 144

So, x, y, 144 are in proportion

x : y :: y : 144

$\frac{x}{y} = \frac{y}{144}$

$y^2 = 144x$ _____ (2)

Putting the value of x in equation (2)

$y^2 = 144 \times \frac{324}{y}$

$y^3 = 144 \times 324$

$y = \sqrt[3]{144 \times 324}$

$y = \sqrt[3]{3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$

$y = \sqrt[3]{6 \times 6 \times 6 \times 6 \times 6 \times 6}$

$y = 6 \times 6$

$y = 36$

Putting y = 36 in equation (1)

$x = \frac{324}{36} = 9$

x = 9, y = 36

2013 - JUNE

[28] For what value of x, the equation $(\log_{\sqrt{x}} 2)^2 = \log_x 2$ is true?

(a) 16 (b) 32
(c) 8 (d) 4 (1 mark)

Answer:

(a) Given

$$\left(\log_{\sqrt{x}^2}\right)^2 = \log_x 2$$

$$\left(\frac{\log 2}{\log \sqrt{x}}\right)^2 = \left(\frac{\log 2}{\log x}\right)$$

$$\left(\frac{\log 2}{\log x^{1/2}}\right)^2 = \frac{\log 2}{\log x}$$

$$\left(\frac{\log 2}{\frac{1}{2}\log x}\right)^2 = \frac{\log 2}{\log x}$$

$$\left(\frac{2\log 2}{\log x}\right)^2 = \left(\frac{\log 2}{\log x}\right)$$

$$4\left(\frac{\log 2}{\log x}\right)^2 = \left(\frac{\log 2}{\log x}\right)^1$$

$$4\,\frac{\log 2}{\log x} = 1$$

$$4 \log 2 = \log x$$

$$\log 2^4 = \log x$$

$$\Rightarrow \quad 2^4 = x \Rightarrow \boxed{x = 16}$$

[29] The mean proportional between 24 and 54 is :

(a) 33 (b) 34
(c) 35 (d) 36 (1 mark)

Answer:

(d) Mean Proportion $= \sqrt{24 \times 54}$

$= \sqrt{1296}$

$= 36$

[30] The triplicate ratio of 4 : 5 is:

(a) 125 : 64 (b) 16 : 25

(c) 64 : 125 (d) 120 : 46 (1 mark)

Answer:

(c) The triplicate Ratio of $4 : 5 = 4^3 : 5^3$

$= 64 : 125$

2013 - DECEMBER

[31] If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}$ then the value of $\left(\frac{a+b+c}{3}\right)^3 = 0$

(a) abc (b) 9abc

(c) $\frac{1}{abc}$ (d) $\frac{1}{9abc}$ (1 mark)

Answer:

(a) If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$

$a^{1/3} + b^{1/3} + c^{1/3} = 0$

$a^{1/3} + b^{1/3} = -c^{1/3}$ ..... (i)

Cube on both side

$(a^{1/3} + b^{1/3})^3 = (-c^{1/3})^3$

$(a^{1/3})^3 + (b^{1/3})^3 + 3 \cdot a^{1/3} \cdot b^{1/3} (a^{1/3} + b^{1/3}) = -c$

$a + b + 3a^{1/3} \cdot b^{1/3} \cdot (-c^{1/3}) = -c$

$a + b - 3a^{1/3} \cdot b^{1/3} \cdot c^{1/3} = -c$

$a + b + c = 3a^{1/3} \cdot b^{1/3} \cdot c^{1/3}$

$\left(\frac{a+b+c}{3}\right) = \frac{3a^{1/3} \cdot b^{1/3} \cdot c^{1/3}}{3}$

$\left(\frac{a+b+c}{3}\right)^3 = (a^{1/3} \cdot b^{1/3} \cdot c^{1/3})^3 = abc$

[32] Find three numbers in the ratio 1 : 2 : 3, so that the sum of their squares is equal to 504

(a) 6, 12, 18

(b) 3, 6, 9

(c) 4, 8, 12

(d) 5, 10, 15 (1 mark)

Answer:

(a) Since Ratio of three Number is 1 : 2 : 3

First No. = x

Second No. = 2x

Third No. = 3x

Sum of squares of numbers = 504

$(x)^2 + (2x)^2 + (3x)^2 = 504$

$x^2 + 4x^2 + 9x^2 = 504$

$14x^2 = 504$

$x^2 = \frac{504}{14}$

$x^2 = 36$

$x = 6$

First No. = x = 6

Second No. = 2x = 2 × 6 = 12

Third No. = 3x = 3 × 6 = 18

[33] The value of $\log_4 9 \cdot \log_3 2$ is:

(a) 3

(b) 9

(c) 2

(d) 1 (1 mark)

Answer:

(d) $\log_4 9 \cdot \log_3 2$

$= \frac{\log 9}{\log 4} \cdot \frac{\log 2}{\log 3}$

$= \frac{\log 3^2}{\log 2^2} \cdot \frac{\log 2}{\log 3}$

$= \frac{2\log 3}{2\log 2} \cdot \frac{\log 2}{\log 3}$

$= 1$

[34] The value of $(\log_y x \cdot \log_z y \cdot \log_x z)^3$ is

(a) 0 (b) - 1

(c) 1 (d) 3 (1 mark)

Answer:

(c) $(\log_y x \cdot \log_z y \cdot \log_x z)^3$

$= \left(\frac{\log x}{\log y} \cdot \frac{\log y}{\log z} \cdot \frac{\log z}{\log x}\right)^3$

$= (1)^3$

$= 1$

[35] Divide 80 into two parts so that their product is maximum, then the numbers are:

(a) 25, 55 (b) 35, 45

(c) 40, 40 (d) 15, 65 (1 mark)

Answer:

(c) The sum of two No. = 80

First No. = x

Second No. = (80 – x)

Product two No = x. (80 – x)

$P = 80x - x^2$ ..... (1)

w.r.f. (x)

$\frac{dp}{dx} = 80 - 2x$ ..... (2)

$\frac{d^2p}{dx^2} = -2$ ..... (3)

For max/minima

$\frac{dp}{dx} = 0$

$80 - 2x = 0$

$2x = 80$

$x = 40$

x = 40 in equation (iii)

$\frac{d^2p}{dx^2} = -2$ (Negative)

function is maximum at x = 40

Numbers are 40, (80 – 40)

= 40, 40

2014 - JUNE

[36] If x : y = 2:3, then (5x+2y):(3x-y)= ___

(a) 19 : 3 (b) 16 : 3

(c) 7 : 2 (d) 7 : 3 (1 mark)

Answer:

(b) Given,

$x : y = 2 : 3$

Let $x = 2k, y = 3k$

$(5x + 2y) : (3x - y)$

$= \frac{(5x + 2y)}{(3x - y)}$

$= \frac{5 \times 2k + 2 \times 3k}{3 \times 2k - 3k}$

$= \frac{10k + 6k}{6k - 3k}$

$= \frac{16k}{3k}$

$= 16 : 3$

[37] If $(25)^{150} = (25x)^{50}$; then the value of x will be :

(a) 5^3 (b) 5^4

(c) 5^2 (d) 5 (1 mark)

Answer:

(b) If $(25)^{150} = (25x)^{50}$

$25^{150} = 25^{50} . x^{50}$

$\Rightarrow \frac{25^{150}}{25^{50}} = x^{50}$

$\Rightarrow 25^{100} = x^{50}$

$\Rightarrow (5^2)^{100} = x^{50}$

$\Rightarrow 5^{200} = x^{50}$

$\Rightarrow (5^4)^{50} = x^{50}$

$\Rightarrow 5^4 = x$

$\Rightarrow x = 5^4$

[38] The value of $\left(\frac{y^a}{y^b}\right)^{a^2+ab+b^2} \times \left(\frac{y^b}{y^c}\right)^{b^2+bc+c^2} \times \left(\frac{y^c}{y^a}\right)^{c^2+ac+a^2}$ is equal to ___ .

(a) y (b) - 1
(c) 1 (d) None of these (1 mark)

Answer:

(c) $\left(\frac{y^a}{y^b}\right)^{a^2+ab+b^2} \left(\frac{y^b}{y^c}\right)^{b^2+bc+c^2} \cdot \left(\frac{y^c}{y^a}\right)^{c^2+ac+a^2}$

$= (y^{a-b})^{a^2+ab+b^2} \cdot (y^{b-c})^{b^2+bc+c^2} \cdot (y^{c-a})^{c^2+ac+a^2}$

$= y^{a^3-b^3} \cdot y^{b^3-c^3} \cdot y^{c^3-a^3}$

$= y^{a^3-b^3+b^3-c^3+c^3-a^3}$

$= y^0 = 1$

[39] If the salary of P is 25% lower than that of Q and the salary of R is 20% higher than that of Q, the ratio of the salary of R and P will be:

(a) 5 : 8 (b) 8 : 5
(c) 5 : 3 (d) 3 : 5 (1 mark)

Answer:

(b) Let Salary of Q= 100

Salary of P = 100 – 25% of 100
= 100 – 25
= 75

Salary of R = 100 + 20% of 100
= 100 + 20
= 120

Ratio of salary of R and P = 120 : 75 = 8 : 5

[40] If $x^2 + y^2 = 7xy$, then $\log \frac{1}{3}(x+y)$ = ___.

(a) (log x+log y)

(b) $\frac{1}{2}$ (log x + log y)

(c) $\frac{1}{3}$ (log x / log y)

(d) $\frac{1}{3}$ (log x + log y) (1 mark)

Answer:

(b) If $x^2 + y^2 = 7xy$

$x^2 + y^2 + 2xy = 7xy + 2xy$

$(x + y)^2 = 9xy$

taking log on both side

$\log (x + y)^2 = \log 9xy$

$2 \log (x + y) = \log 9 + \log x + \log y$

$2 \log (x + y) = \log 3^2 + \log x + \log y$

$2 \log (x + y) = 2 \log 3 + \log x + \log y$

$2 \log (x + y) - 2 \log 3 = \log x + \log y$

$2 \left[\log \frac{(x + y)}{3}\right] \left[\log \frac{(x + y)}{3}\right]$

$= \log x + \log y$

$\log \frac{(x + y)}{3} = \frac{1}{2} [\log x + \log y]$

[41] A person has assets worth ₹ 1,48,200. He wish to divide it amongst his wife, son and daughter in the ratio 3 : 2 : 1 respectively. From this assets, the share of his son will be:

(a) ₹ 24,700 (b) ₹ 49,400

(c) ₹ 74,100 (d) ₹ 37,050 (1 mark)

Answer:

(b) A person has Assets worth = ₹ 1,48,200

Ratio of share of wife, son & daughter

$= 3 : 2 : 1$

Sum of Ratio $= 3 + 2 + 1 = 6$

Share of Son $= \frac{2}{6} \times 1,48,200$

$= 49,400$

[42] If $x = \log_{24}12$, $y = \log_{36}24$ and $z = \log_{48}36$, then $xyz + 1 =$ ___

(a) 2xy (b) 2xz

(c) 2yz (d) 2 (1 mark)

Answer:

(c) If $x = \log_{24}12$, $y = \log_{36}24$ and $z = \log_{48}36$ then

XYZ + 1

$= \log_{24}12 \times \log_{36}24 \times \log_{48}36 + 1$

$= \frac{\log 12}{\log 24} \cdot \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48} + 1$

$= \frac{\log 12}{\log 48} + 1$

$= \frac{\log 12 + \log 48}{\log 48}$

$= \frac{\log(12 \times 48)}{\log 48}$

$= \frac{\log(576)}{\log 48}$

$= \frac{\log 24^2}{\log 48}$

$= \frac{2\log 24}{\log 48}$

$= 2 \cdot \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48}$

$= 2.\log_{36}24 \cdot \log_{48}36$

$= 2\ y\ z$

2014 - DECEMBER

[43] If $\log x = a + b$, $\log y = a - b$ then the value of $\log \frac{10x}{y^2}$ = _____.

(a) $1 - a + 3b$ (b) $a - 1 + 3b$

(c) $a + 3b + 1$ (d) $1 - b + 3a$ (1 mark)

Answer:

(a) Given $\log x = a + b$, $\log y = a - b$

$\log\left(\frac{10x}{y^2}\right) = \log 10x - \log y^2$

$= \log 10 + \log x - 2\log y$

$= 1 + (a + b) - 2(a - b)$

$= 1 + a + b - 2a + 2b$

$= 1 - a + 3b$

[44] If $x = 1 + \log_p qr$, $y = 1 + \log_q rp$ and $z = 1 + \log_r pq$ then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$ _____

(a) 0 (b) 1

(c) -1 (d) 3 (1 mark)

Answer:

(b) If $x = 1 + \log_p qr$, $y = 1 + \log_q rp$, $z = 1 + \log_r pq$

$$x = 1 + \frac{\log qr}{\log p}$$

$$x = \frac{\log p + \log qr}{\log p}$$

$$x = \frac{\log pqr}{\log p}$$

$$\frac{1}{x} = \frac{\log p}{\log pqr}$$

Similarly

$$\frac{1}{y} = \frac{\log q}{\log pqr}$$

$$\frac{1}{z} = \frac{\log r}{\log pqr}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{\log p}{\log pqr} + \frac{\log q}{\log pqr} + \frac{\log r}{\log pqr}$$

$$= \frac{\log p + \log q + \log r}{\log pqr}$$

$$= \frac{\log pqr}{\log pqr}$$

$$= 1$$

[45] For three months, the salary of a person are in the ratio 2 : 4 : 5. If the difference between the product of salaries of the first two months and last two months is ₹ 4,80,00,000; then the salary of the person for the second month will be:

(a) ₹ 4,000 (b) ₹ 6,000

(c) ₹ 8,000 (d) ₹ 12,000 (1 mark)

Answer:

(c) Ratio of the salary of a person in three months = 2 : 4 : 5

Let, Salary of Ist month = 2x
Salary of IInd month = 4x
Salary of IIIrd month = 5x

Given

(Salary of Product of last two months) – (Salary of Product Ist two months)

$= 4,80,00,000$

$(4x.5x) – (2x.4x) = 4,80,00,000$

$20x^2 – 8x^2 = 4,80,00,000$

$12x^2 = 4,80,00,000$

$x^2 = 40,00,000$

$x = 2,000$

Salary of the person for second month = $4x = 4 \times 2,000 = 8,000$

2015 - JUNE

[46] A dealer mixes rice costing ₹ 13.84 per Kg. with rice costing ₹ 15.54 and sells the mixture at ₹ 17.60 per Kg. So, he earns a profit of 14.6% on his sale price. The proportion in which he mixes the two qualities of rice is:

(a) 3 : 7 (b) 5 : 7
(c) 7 : 9 (d) 9 : 11 (1 mark)

Answer:

(a) Let SP of mixture is ₹ 100

Then Profit = 14.6% of 100
= 14.6

CP of mixture = (100 – 14.6)
= 85.4

$\because$ If SP is ₹ 100 then CP = 85.4

$\therefore$ If SP is ₹ 1 then CP = $\frac{85.4}{100}$

$\therefore$ If SP is ₹ 17.60 then CP $= \frac{85.4}{100} \times 17.60$

$= 15.0304$

$\therefore$ CP of the Mixture per kg = ₹ 15.0304

2^{nd} difference = Profit by SP 1 kg of 2^{nd} kind @ ₹ 15.0304

$= 15.54 - 15.0304$

$= 0.5096$

1^{st} difference = ₹ 15.0304 – 13.84

= ₹ 1.1904

The Require Ratio = (2^{nd} difference) : (1^{st} difference)

= 0.5096 : 1.1904

= 3 : 7

[47] If $p^x = q$, $q^y = r$ and $r^z = p^6$, then the value of xyz will be:

(a) 0 (b) 1

(c) 3 (d) 6 (1 mark)

Answer:

(d) If $p^x = q$, $q^y = r$ and $r^z = p^6$

$q = p^x$, $q^y = r$ and $r^z = p^6$

$(q^y)^z = p^6$

$[(p^x)^y]^z = p^6$

$p^{xyz} = p^6 = xyz = 6$

[48] If log x = m + n and log y = m - n, then log $(10x/y^2)$ =

(a) 3n – m + 1 (b) 3m – n + 1

(c) 3n + n + 1 (d) 3m + n + 1 (1 mark)

Answer:

(a) Log x = m + n and log y = m – n

Then $\log \left(\frac{10x}{y^2}\right) = \log 10x - \log y^2$

$= \log 10 + \log x - 2 \log y$

$= 1 + \log x - 2 \log y$

$= 1 + (m + n) - 2 (m - n)$

$= 1 + m + n - 2m + 2n$

$= 3n - m + 1$

[49] If $15(2p^2 - q^2) = 7pq$, where p and q are positive, then p : q will be:

(a) 5 : 6 (b) 5 : 7

(c) 3 : 5 (d) 8 : 3 (1 mark)

Answer:

(a) If $15(2p^2 - q^2) = 7pq$

$30p^2 - 15q^2 = 7pq$

$30p^2 - 7pq - 15q^2 = 0$

$30p^2 - 25pq + 18pq - 15q^2 = 0$

$5p(6p - 5q) + 3q(6p - 5q) = 0$

$(6p - 5q)(5p + 3q) = 0$

If $6p - 5q = 0$ and $5p + 3q = 0$

$6p = 5q \quad 5p = -3q$

$\frac{p}{q} = \frac{5}{6} = p : q = 5 : 6 \quad \frac{p}{q} = \frac{-3}{5}$

(not possible)

2015 - DECEMBER

[50] The ratio of third proportion of 12, 30 to the mean proportion of 9, 25 is:

(a) 2:1

(b) 5:1

(c) 7:15

(d) 3:5 (1 mark)

Answer:

(b) The third proportion of 12,30

$c = \frac{b^2}{a} = \frac{(30)^2}{12} = \frac{900}{12} = 75$

The Mean proportion of 9,25

$b = \sqrt{ac} = \sqrt{9 \times 25} = \sqrt{225} = 15$

Ratio of third proportion of 12, 30 and Mean proportion of 9, 25 = 75:15

= 5:1

[51] The value of $\log_5 3 \times \log_3 4 \times \log_2 5$.

(a) 0 (b) 1

(c) 2 (d) $\frac{1}{2}$ (1 mark)

Answer:

(c) $\log_5 3 \times \log_3 4 \times \log_2 5$

$= \frac{\log 3}{\log 5} \times \frac{\log 4}{\log 3} \times \frac{\log 5}{\log 2}$

$= \frac{\log 4}{\log 2}$

$= \frac{\log 2^2}{\log 2}$

$= \frac{2 \log 2}{\log 2} = 2$

[52] What number must be added to each of the numbers 10, 18, 22, 38 to make the numbers is proportion?

(a) 2 (b) 4

(c) 8 (d) None of these. (1 mark)

Answer:

(a) Let x to be added

Then (10 + x), (18 + x), (22 + x), (38 + x) are in prop.

Product of Extremes = Product of Mean

$(10 + x)(38 + x) = (18 + x)(22 + x)$

$380 + 10x + 38x + x^2 = 396 + 18x + 22x + x^2$

$48x + 380 = 396 + 40x$

$48x - 40x = 396 - 380$

$8x = 16$

$x = 2$

[53] The value of $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$ is :

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) 2 (1 mark)

Answer:

(b) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{2^n + 2^n.2^{-1}}{2^n.2^1 - 2^n}$

$= \frac{2^n(1 + 2^{-1})}{2^n(2^1 - 1)}$

$= \frac{\left(\frac{1}{1} + \frac{1}{2}\right)}{(2 - 1)}$

$= \frac{\left(\frac{2 + 1}{2}\right)}{1}$

$= \left(\frac{3}{2}\right)$

2016 - JUNE

[54] The integral part of a logarithm is called _____ and the decimal part of a logarithm is called _____.

(a) Mantissa, Characteristic (b) Characteristic, Mantissa

(c) Whole, Decimal (d) None of these. (1 mark)

Answer:

(b) The integral part of a logarithms is called **Characteristic** and the decimal part of a logarithm is called **mantissa**.

[55] The value of $\left[\frac{x^2 - (y - z)^2}{(x + z)^2 - y^2} + \frac{y^2 - (x - z)^2}{(x + y)^2 - z^2} + \frac{z^2 - (x - y)^2}{(y + z)^2 - x^2}\right]$ is

(a) 0 (b) 1

(c) – 1 (d) ∞ (1 mark)

Answer:

(b) $\frac{x^2 - (y - z)^2}{(x + z)^2 - y^2} + \frac{y^2 - (x - z)^2}{(x + y)^2 - z^2} + \frac{z^2 - (x - y)^2}{(y + z)^2 - x^2}$

$= \frac{(x + y - z)(x - y + z)}{(x + z + y)(x + z - y)} + \frac{(y + x - z)(y - x + z)}{(x + y + z)(x + y - z)} + \frac{(z + x - y)(z - x + y)}{(y + z + x)(y + z - x)}$

$$= \frac{x+y-z}{x+y+z} + \frac{y+z-x}{x+y+z} + \frac{z+x-y}{x+y+z}$$

$$= \frac{x+y-z+y+z-x+z+x-y}{x+y+z}$$

$$= \frac{x+y+z}{x+y+z} = 1$$

[56] X, Y, Z together starts a business. If X invests 3 times as much as Y invests and Y invests two third of what Z invests, then the ratio of capitals of X, Y, Z is:

(a) 3:9:2 (b) 6:3:2

(c) 3:6:2 (d) 6:2:3 (1 mark)

Answer:

(d) Given $x = 3y$ and $y = \frac{2}{3}z$

$$\frac{x}{y} = \frac{3}{1} \text{ and } \frac{y}{z} = \frac{2}{3}$$

$x : y = 3 : 1$ and $y : z = 2 : 3$

$= 3 \times 2 : 1 \times 2$

$= 6 : 2$

$x : y : z = 6 : 2 : 3$

[57] If $\log_4(x^2 + x) - \log_4(x + 1) = 2$, then the value of X is:

(a) 2 (b) 3

(c) 16 (d) 8 (1 mark)

Answer:

(c) If $\log_4 (x^2 + x) - \log_4 (x + 1) = 2$

$$\Rightarrow \log_4 \left\{\frac{(x^2+x)}{(x+1)}\right\} = 2$$

$$\Rightarrow \log_4 \left\{\frac{x(x+1)}{(x+1)}\right\} = 2$$

$$\Rightarrow \log_4 x = 2$$

$$x = 4^2$$

$$x = 16$$

[58] Value of $\frac{1}{\log_3^{60}}+\frac{1}{\log_4^{60}}+\frac{1}{\log_5^{60}}$ is :

(a) 0 (b) 1

(c) 5 (d) 60 (1 mark)

Answer:

(b) $\frac{1}{\log_3 60}+\frac{1}{\log_4 60}+\frac{1}{\log_5 60}$

$= \log_{60} 3 + \log_{60} 4 + \log_{60} 5 \quad \because \left[\frac{1}{\log_a b} = \log_b a\right]$

$= \log_{60}(3\times 4\times 5)$

$= \log_{60} 60$

$= 1$

2016 - DECEMBER

[59] If $3^x = 5^y = 75^z$, then

(a) $x + y - z = 0$ (b) $\frac{2}{x}+\frac{1}{y}=\frac{1}{z}$

(c) $\frac{1}{x}+\frac{2}{y}=\frac{1}{z}$ (d) $\frac{2}{x}+\frac{1}{z}=\frac{1}{y}$ (1 mark)

Answer:

(c) If $3^x = 5^y = 75^z = k$ (let)

then $3^x = k, 5^y = k, 75^z = k$

$3 = k^{1/x}, 5 = k^{1/y}, 75 = k^{1/z}$

we know that

$75 = 3 \times 5 \times 5$

$k^{\frac{1}{z}} = k^{\frac{1}{x}} . k^{\frac{1}{y}} . k^{\frac{1}{y}}$

$k^{\frac{1}{z}} = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{y}}$

on comparing

$$\frac{1}{z} = \frac{1}{x} + \frac{1}{y} + \frac{1}{y}$$

$$\frac{1}{z} = \frac{1}{x} + \frac{2}{y}$$

$$\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$$

[60] If log 2 = 0.3010 and log 3 = 0.4771, then the value of log 24 is:

(a) 1.0791 (b) 1.7323

(c) 1.3801 (d) 1.8301 (1 mark)

Answer:

(c) If log 2 = 0.3010 and log 3 = 0.4771

then log 24 = log (2 × 2 × 2 × 3)

= log 2 + log 2 + log 2 + log 3

= 3 log 2 + log 3

= 3 × 0.3010 + 0.4771

= 0.9030 + 0.4771

= 1.3801

[61] If abc = 2, then the value of $\frac{1}{1 + a + 2b^{-1}} + \frac{1}{1 + \frac{1}{2}b + c^{-1}} + \frac{1}{1 + c + a^{-1}}$ is:

(a) 1 (b) 2

(c) 3 (d) $\frac{1}{2}$ (1 mark)

Answer:

(a) If abc = 2

$ab = \frac{2}{c} = 2c^{-1}$ $\quad a = \frac{2}{bc} = 2b^{-1}c^{-1}$

$bc = \frac{2}{a} = 2a^{-1}$ $\quad b = \frac{2}{ca} = 2c^{-1}a^{-1}$

$ca = \frac{2}{b} = 2b^{-1}$ $\quad c = \frac{2}{ab} = 2a^{-1}b^{-1}$

Given $\frac{1}{1+a+2b^{-1}}+\frac{1}{1+\frac{1}{2}b+c^{-1}}+\frac{1}{1+c+a^{-1}}$

$= \frac{1}{1+a+2b^{-1}}+\frac{2b^{-1}}{2b^{-1}(1+\frac{1}{2}b+c^{-1})}+\frac{a}{a(1+c+a^{-1})}$

$= \frac{1}{(1+a+2b^{-1})}+\frac{2b^{-1}}{2b^{-1}+1+2b^{-1}c^{-1}}+\frac{a}{a+ac+1}$

$= \frac{1}{1+a+2b^{-1}}+\frac{2b^{-1}}{2b^{-1}+1+a}+\frac{a}{a+2b^{-1}+1}$

$= \frac{1+2b^{-1}+a}{1+a+2b^{-1}}$

$= 1$

[62] There are total 23 coins of ₹ 1, ₹ 2 and ₹ 5 in a bag. If their value is ₹ 43 and the ratio of coins of ₹ 1 and ₹ 2 is 3:2. Then the number of coins of ₹ 1 is:

(a) 12 (b) 5

(c) 10 (d) 14 (1 mark)

Answer:

(a) Total no. of coins = 23

Ratio of ₹ 1 coin : ₹ 2 coins = 3 : 2

let No. of ₹ 1 coins = 3x

No. of ₹ 2 coins = 2x

No. of ₹ 5 coins = 23 – 3x – 2x

= 23 – 5x

Total value of all coins = 43

$3x \times 1 + 2x \times 2 + (23 - 5x)\,5 = 43$

$3x + 4x + 115 - 25x = 43$

$-18x = 43 - 115$

$-18x = -72$

$x = \frac{-72}{-18} = 4$

No. of ₹ 1 coins = $3x = 3 \times 4 = 12$

2017 - JUNE

[63] If a : b = 2 : 3, b : c = 4 : 5 and c : d = 6 : 7, then a : d is:

(a) 24 : 35 (b) 8 : 15

(c) 16 : 35 (d) 7 : 15 (1 mark)

Answer:

(c) $a : b = 2 : 3 \Rightarrow \frac{a}{b} = \frac{2}{3}$ _____(i)

$b : c = 4 : 5 \Rightarrow \frac{b}{c} = \frac{4}{5}$ _____(ii)

$c : d = 6 : 7 \Rightarrow \frac{c}{d} = \frac{6}{7}$ _____(iii)

Multiply equation (i) & (ii) & (iii)

$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} = \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{16}{35}$

[64] The value of $\log (1^3 + 2^3 + 3^3 + ...n^3)$ is equal to:

(a) 3 log 1 + 3 log 2 + ... + 3 log n

(b) 2 log n + 2 log (n+1) – 2 log 2

(c) log n + log (n+1) + log (2n+1) – log 6 (1 mark)

(d) 1

Answer:

(b) $\log (1^3 + 2^3 + 3^3 + ------+ n^3)$

$= \log (\Sigma n^3)$

$= \log \left[\frac{n(n+1)}{2}\right]^2$

$= 2 \log \left[\frac{n(n+1)}{2}\right]$

$= 2 [\log n + \log (n + 1) - \log 2]$

$= 2 \log n + 2 \log (n + 1) - 2 \log 2$

[65] If $a = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ and $b = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ then the value of $\frac{1}{a^2} + \frac{1}{b^2}$ is equal to:

(a) 480 (b) 482

(c) 484 (d) 486 (1 mark)

Answer:

(b) If $a = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ and $b = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

$$a + b = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} + \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$$

$$= \frac{(\sqrt{6}+\sqrt{5})^2 + (\sqrt{6}-\sqrt{5})^2}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})}$$

$$= \frac{6 + 5 + 2\sqrt{30} + 6 + 5 - 2\sqrt{30}}{(\sqrt{6})^2 - (\sqrt{5})^2}$$

$$= \frac{22}{6-5} = \frac{22}{1} = 22$$

$$a \cdot b = \left(\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}\right)\left(\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right) = 1$$

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{b^2 + a^2}{a^2 b^2} = \frac{(a+b)^2 - 2ab}{(ab)^2}$$

$$= \frac{(22)^2 - 2 \times 1}{(1)^2} = \frac{484 - 2}{1} = 482$$

2017 - DECEMBER

[66] The ratio of the number of ₹ 5 coins and ₹ 10 coins is 8 : 15. If the value of ₹ 5 coins is ₹ 360, then the number of ₹ 10 coins will be:

(a) 72 (b) 120

(c) 135 (d) 185 (1 mark)

Answer:

(c) Ratio of ₹ 5 coins and ₹ 10 coins = 8 : 15

Let the No. of ₹ 5 coins = 8x

and the No. of ₹ 10 coins = 15x

The value of ₹ 5 coins = ₹ 5 × 8x

$$360 = 40x$$

$$x = \frac{360}{40}$$

$$x = 9$$

No. of ₹ 10 coins = 15x
= 15 × 9
= 135

[67] If $\log_3 [\log_4 (\log_2 x)] = 0$, then the value of 'x' will be:
(a) 4 (b) 8
(c) 16 (d) 32 (1 mark)

Answer:

(c) If $\log_3 [\log_4 (\log_2 x)] = 0$

$\log_4(\log_2 x) = 3^0$ [$\because \log_a b = x \Rightarrow b = a^x$]

$\log_4 (\log_2 x) = 1$

$\log_2 x = 4^1$

$\log_2 x = 4$

$x = 2^4$

$x = 16$

[68] If $\log\left(\frac{x-y}{2}\right) = \frac{1}{2}(\log x + \log y)$, then the value of $x^2 + y^2 =$ _____.
(a) 2xy (b) 4xy
(c) $2x^2y^2$ (d) 6xy (1 mark)

Answer:

(d) If $\log\left(\frac{x-y}{2}\right) = \frac{1}{2}(\log x + \log y)$

$2\log\left(\frac{x-y}{2}\right) = \log x + \log y$

$\log\left(\frac{x-y}{2}\right)^2 = \log (xy)$

$\Rightarrow \left(\frac{x-y}{2}\right)^2 = xy$

$\Rightarrow \left(\frac{x-y}{4}\right)^2 = xy$

$\Rightarrow x^2 + y^2 - 2xy = 4xy$

$\Rightarrow x^2 + y^2 = 4xy + 2xy$

$\Rightarrow x^2 + y^2 = 6xy$

[69] If $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{x}$ are in proportion, then the value of 'x' will be:

(a) $\frac{15}{2}$ (b) $\frac{6}{5}$

(c) $\frac{10}{3}$ (d) $\frac{5}{6}$ (1 mark)

Answer:

(a) If $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{x}$ are in proportion

then, product of extremes = Product of means

$\frac{1}{2} \times \frac{1}{x} = \frac{1}{3} \times \frac{1}{5}$

$\frac{1}{2x} = \frac{1}{15}$

$2x = 15$

$x = 15/2$

2018 - MAY

[70] If p : q is the sub-duplicate ratio of $p - x^2 : q - x^2$, then x^2 is:

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$

(c) $\frac{qp}{p-q}$ (d) None. (1 mark)

Answer:

(d) Sub duplicate ratio of $(p - x^2) : (q - x^2) = \sqrt{p-x^2} : \sqrt{q-x^2}$

$p:q = \sqrt{p-x^2} : \sqrt{q-x^2}$

$\frac{P}{q} = \frac{\sqrt{p-x^2}}{\sqrt{q-x^2}}$

an squaring both side

$\frac{p^2}{q^2} = \frac{p-x^2}{q-x^2}$

$p^2 (q - x^2) = q^2(p - x^2)$
$p^2q - p^2x^2 = q^2p - q^2x^2$
$p^2q - q^2p = p^2x^2 - q^2x^2$
$pq (p - q) = (p^2 - q^2)x^2$
$pq (p - q) = (p + q)(p - q)x^2$
$x^2 = \frac{pq(p-q)}{(p+q)(P-q)}$
$x^2 = \frac{pq}{(p+q)}$

[71] The value of the expression :

$a^{\log_a b \cdot \log_b^c \cdot \log_c^d \cdot \log_d t}$

(a) t (b) abcdt
(c) (a + b + c + d + t) (d) None (1 mark)

Answer:

(a) $a^{\log_a^b \cdot \log_b^c \cdot \log_c^d \cdot \log_d^t}$

$= a^{\frac{\log^b}{\log^a} \cdot \frac{\log^c}{\log^b} \cdot \frac{\log^d}{\log^c} \cdot \frac{\log^t}{\log^d}}$

$= a^{\frac{\log^t}{\log^a}}$

$= a^{\log_a^t}$ $[\because e^{\log_e^x} = x]$

$= t$

[72] The mean proportional between 24 and 54 is:

(a) 33 (b) 34
(c) 35 (d) 36 (1 mark)

Answer:

(d) Mean proportion $b = \sqrt{ac}$

$= \sqrt{24 \times 54}$

$= \sqrt{1,296}$

$= 36$

[73] The value of $\log_4 9 . \log_3 2$ is:

(a) 3 (b) 2
(c) 9 (d) 1 (1 mark)

Answer:

(d) $\log_4 9 . \text{Log}_3 2 = \frac{\log 9}{\log 4} . \frac{\log 2}{\log 3}$

$= \frac{\log 3^2}{\log 2^2} . \frac{\log 2}{\log 3}$

$= \frac{2\log 3}{2\log 2} . \frac{\log 2}{\log 3}$

$= 1$

[74] $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(a) $\frac{1}{2}$

(b) $\frac{3}{2}$

(c) $\frac{2}{3}$

(d) $\frac{1}{3}$ (1 mark)

Answer:

(b) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{2^n + 2^n . 2^{-1}}{2^n . 2^{+1} - 2^n}$

$= \frac{2^n (1 + 2^{-1})}{2^n . (2 - 1)}$

$= \frac{\left(1 + \frac{1}{2}\right)}{1}$

$= \frac{\frac{3}{2}}{1}$

$= \frac{3}{2}$

2018 - NOVEMBER

[75] $\frac{3x-2}{5x+6}$ is the duplicate ratio of $\frac{2}{3}$ then find the value of x:

(a) 2 (b) 6

(c) 5 (d) 9 (1 mark)

Answer:

(b) $\because$ $\frac{3x-2}{5x+6}$ is the duplicate ratio of $\frac{2}{3}$

i.e. $\frac{3x-2}{5x+6} = \frac{2^2}{3^2}$

$\Rightarrow$ $\frac{3x-2}{5x+6} = \frac{4}{9}$

$27x - 18 = 20x + 24$

$27x - 20x = 24 + 18$

$7x = 42$

$X = 6$

[76] $\frac{2^{m+1} \times 3^{2m-n+3} \times 5^{n+m+4} \times 6^{2n+m}}{6^{2m+n} \times 10^{n+1} \times 15^{m+3}}$

(a) 3^{2m-2n} (b) 3^{2n-2m}

(c) 1 (d) None of the above (1 mark)

Answer:

(c) $\frac{2^{m+1} \times 3^{2m-n+3} \times 5^{n+m+4} \times 6^{2n+m}}{6^{2m+n} \times 10^{n+1} \times 15^{m+3}}$

$= \frac{2^{m+1} \times 3^{2m-n+3} \times 5^{n+m+4} \times (2\times 3)^{2n+m}}{(2\times 3)^{2m+n} \times (2\times 5)^{n+1} \times (3\times 5)^{m+3}}$

$= \frac{2^{m+1} \times 3^{2m-n+3} \times 5^{n+m+4} \times 2^{2n+m} \times 3^{2n+m}}{2^{2m+n} \times 3^{2m+n} \times 2^{n+1} \times 5^{n+1} \times 3^{m+3} \times 5^{m+3}}$

$= \frac{2^{m+1+2n+m} \times 3^{2m-n+3+2n+m} \times 5^{n+m+4}}{2^{2m+n+n+1} \times 3^{2m+n+m+3} \times 5^{n+1+m+3}}$

$= \frac{2^{2m+2n+1} \times 3^{3m+n+3} \times 5^{m+n+4}}{2^{2m+2n+1} \times 3^{3m+n+3} \times 5^{m+n+4}} = 1$

[77] If x : y : z = 7 : 4 : 11 then $\frac{x + y + z}{z}$ is:

(a) 2 (b) 3
(c) 4 (d) 5 (1 mark)

Answer:

(a) If x: y: z = 7: 4: 11

Let x = 7k, y = 4k, z = 11k

$$\frac{x + y + z}{2} = \frac{7k + 4k + 11k}{11k} = \frac{22k}{11k} = 2$$

[78] $\log_2 \log_2 \log_2 16$ =?

(a) 0 (b) 3
(c) 1 (d) 2 (1 mark)

Answer:

(c) $\log_2 \log_2 \log_2^{16}$

$= \log_2 \log_2 (\log_2^{24})$

$= \log_2 \log_2^4 \log_2^2$

$= \log_2 \log_2^4$ ($\because \log_2^2 = 1$)

$= \log_2 \log_{2^2}^2$

$= \log_2^2 . \log_2^2$

$= 1 \times 1$

$= 1$

2019 - JUNE

[79] If the ratio of two numbers is 7 : 11. If 7 is added to each number then the new ratio will be 2 : 3 then the numbers are.

(a) 49, 77
(b) 42, 45
(c) 43, 42
(d) 39, 40 (1 mark)

Answer:

(a) Ratio of two Numbers = 7 : 11

Let Ist No = 7x

IInd No = 11x

Given Condition

$(7x + 7) : (11x + 7) = 2 : 3$

$$\frac{7x+7}{11x+7} = \frac{2}{3}$$

$21x + 21 = 22x + 14$

$21 - 14 = 22x - 21x$

$7 = x$

Ist No = $7x = 7 \times 7 = 49$

IInd No = $11x = 11 \times 7 = 77$

[80] If $2^{x^2} = 3^{y^2} = 12^{z^2}$ then

(a) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$

(b) $\frac{1}{x^2} + \frac{2}{y^2} = \frac{1}{z^2}$

(c) $\frac{2}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$

(d) None (1 mark)

Answer:

(c) If $2^{x^2} = 3^{y^2} = 12^{z^2} = K$

$2^{x^2} = K, 3^{y^2} = K, 12^{z^2} = K$

$2 = K^{\frac{1}{x^2}}, 3 = K^{\frac{1}{y^2}}, 12 = K^{\frac{1}{z^2}}$

Now,

$12 = 2 \times 2 \times 3$

$K^{\frac{1}{z^2}} = K^{\frac{1}{x^2}} \times K^{\frac{1}{x^2}} \times K^{\frac{1}{y^2}}$

$K^{\frac{1}{z^2}} = K^{\frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{y^2}}$

On comparing

$$\frac{1}{z^2} = \frac{1}{x^2} + \frac{1}{x^2} + \frac{1}{y^2}$$

$$\frac{1}{z^2} = \frac{2}{x^2} + \frac{1}{y^2}$$

[81] The value of

$$\log_5\left(1+\frac{1}{5}\right) + \log_5\left(1+\frac{1}{6}\right) + \text{----------} + \log_5\left(1+\frac{1}{624}\right)$$

(a) 2
(b) 3
(c) 5
(d) 0 (1 mark)

Answer:

(b) If $\log_5\left(1+\frac{1}{5}\right) + \log_5\left(1+\frac{1}{6}\right) + \ldots\ldots\ldots\ldots + \log_5\left(1+\frac{1}{624}\right)$

$= \log\left(\frac{6}{5}\right) + \log\left(\frac{7}{6}\right) \log\left(\frac{8}{7}\right) + \ldots\ldots\ldots\ldots + \log\left(\frac{625}{624}\right)$

$= \log_5\left(\frac{\cancel{6}}{5}\times\frac{\cancel{7}}{\cancel{6}}\times\frac{\cancel{8}}{\cancel{7}}\times\ldots\ldots\ldots\frac{\cancel{624}}{623}\times\frac{625}{\cancel{624}}\right)$

$= \log_5\left(\frac{625}{5}\right)$

$= \log_5(125) = \log_5 5^3 = 3\log_5 5$

$= 3 \times 1$

$= 3$

[82] $\log_{2\sqrt{2}}(512) : \log_{3\sqrt{2}} 324 =$

(a) 128 : 81
(b) 2 : 3
(c) 3 : 2
(d) None (1 mark)

Answer:

(c) $\log_{2\sqrt{2}} 512 : \log_{3\sqrt{2}} 324$

$= \frac{\log 512}{\log 2\sqrt{2}} : \frac{\log 324}{\log 3\sqrt{2}}$

$$= \frac{\log (8)^3}{\log \sqrt{2\times2\times2}} : \frac{\log 18^2}{\log \sqrt{3\times3\times2}}$$

$$= \frac{\log (8)^3}{\log (8)^{1/2}} : \frac{\log (18)^2}{\log (18)^{1/2}}$$

$$= \frac{3 \log 8}{1/2 \log 8} : \frac{2 \log 18}{1/2 \log 18}$$

$$(3 \times 2) : (2 \times 2)$$

$$= 6 : 4$$

$$= 3 : 2$$

[83] If $P = x^{1/3} + x^{-1/3}$ then $P^3 - 3P =$

(a) 3

(b) $\frac{1}{2}\left(x + \frac{1}{x}\right)$

(c) $\left(x + \frac{1}{x}\right)$

(d) $2\left(x + \frac{1}{x}\right)$ (1 mark)

Answer:

(c) If $P = x^{1/3} + n^{-1/p}$ then $P^3 - 3P$

Given $P = x^{1/3} + x^{-1/p}$ .....(1)

Cube on both side

$P^3 = (x^{1/3} + x^{-1/p})^3$

$P^3 = (x^{1/3})^3 + (x^{-1/p})^3 + 3x^{1/3} \cdot x^{-1/3} (x^{1/3} + x^{-1/3})$

$= x + x^{-1} + 3 \times 1 \times P$

$P^3 = x + \frac{1}{x} + 3P$

$P^3 - 3P = x + \frac{1}{x}$

2019 - NOVEMBER

[84] The ratio of two numbers are 3 : 4. The difference of their squares is 28 Greater no. is:

(a) 8

(b) 12

(c) 24

(d) 64. (1 mark)

Answer:

(a) Let the two numbers be x and y

Greater no. ⇒ y

Smaller no. ⇒ x

According to question,

$\frac{x}{y} = \frac{3}{4}$ — Eq 1 and $y^2 - x^2 = 28$ — Eq 2

Further solving Eq 1

$x = \frac{3}{4}y$ — Eq 3

Put Eq 3 in Eq 2

$y^2 - \left(\frac{3}{4}y\right)^2 = 28$

$\frac{y^2}{1} - \frac{9y^2}{16} = 28$

$\frac{7y^2}{16} = 28$

$y^2 = \frac{28 \times 16}{7}$

$y^2 = 64$

$\Rightarrow y = 8$ {square root both sides}

So, the greater number i.e. y is equal to 8.

[85] The price of scooter and moped are in the ratio 7 : 9. The price of moped is ₹ 1,600 more than that of scooter. Then the price of moped is:

(a) ₹ 7,200
(b) ₹ 5,600
(c) ₹ 800
(d) ₹ 700 (1 mark)

Answer:

(a) $\frac{\text{Price of scooter}}{\text{Price of Moped}} = \frac{7}{9}$

Let; the price of scooter = 7x
and price of moped = 9x
According to question

$9x = 7x + 1600$
$\Rightarrow 2x = 1600$
$\Rightarrow x = ₹\ 800$

So, price of moped = 9x = 9 (800) = ₹ 7,200

[86] $\log_{0.01} 10{,}000 = ?$

(a) 2
(b) –2
(c) 4
(d) –4 (1 mark)

Answer:

(b) $\log_{0.01} 10{,}000$

$\Rightarrow \frac{\log 10{,}000}{\log 0.01}$ Since $\log_a b = \frac{\log b}{\log a}$

$\Rightarrow \frac{\log (10)^4}{\log\left(\frac{1}{100}\right)}$ $\because \log a^n = n \log a$

$\Rightarrow \frac{4 \times \log 10}{\log 1 - \log 100}$ $\because \log\left(\frac{b}{a}\right) = \log b - \log a$

$\Rightarrow \frac{4 \times 1}{0 - \log (10)^2}$ $\log 10 = 1$, $\log 1 = 0$

$\Rightarrow \frac{4}{-2 \log 10} = \frac{4}{-2 \times 1} = -2$

[87] Value of $\left[9^{n+\frac{1}{4}} \cdot \frac{\sqrt{3.3^n}}{3.\sqrt{3^{-n}}}\right]^{\frac{1}{n}}$

(a) 9
(b) 27
(c) 81
(d) 3 (1 mark)

Answer:

(b) $= \left[\frac{9^{n+\frac{1}{4}} \sqrt{3^{(n+1)}}}{3.\sqrt{3^{-n}}}\right]^{\frac{1}{n}}$

$= \left[\frac{3^{2n+\frac{1}{2}} . 3^{\frac{(n+1)}{2}}}{3.\ 3^{-n/2}}\right]^{\frac{1}{n}}$

$= \left[\frac{3^{2n+\frac{1}{2}+\frac{n}{2}+\frac{1}{2}}}{3^{1-n/2}}\right]^{\frac{1}{n}}$

$= \left[(3)^{\frac{5n}{2}+1-1+\frac{n}{2}}\right]^{\frac{1}{n}}$

$= \left[(3)^{\frac{6n}{2}}\right]^{\frac{1}{n}}$

$= (3)^3$

$= 27$

[88] If $x = \sqrt{3}+\frac{1}{\sqrt{3}}$ then $\left(x-\frac{\sqrt{126}}{\sqrt{42}}\right)\left(x-\frac{1}{x-\frac{2\sqrt{3}}{3}}\right)$ =?

(a) 5/6
(b) 6/5
(c) 2/3
(d) – 3/5 (1 mark)

Answer:

(a) $x = \sqrt{3} + \frac{1}{\sqrt{3}}$----------Equation (1)

$= (x - \sqrt{3}) = \frac{1}{\sqrt{3}}$ --------- Equation (2) $\left(x - \frac{1}{\sqrt{3}}\right) = \sqrt{3}$ — Equation (3)

$\left(x \frac{\sqrt{126}}{\sqrt{42}}\right)\left(x \frac{-1}{\left(x - \frac{2\sqrt{3}}{\sqrt{3}}\right)}\right)$

$\left(x \frac{-3\sqrt{14}}{\sqrt{3} \times \sqrt{14}}\right)\left(x \frac{-1}{\frac{x-1}{\sqrt{3}} \frac{-1}{\sqrt{3}}}\right)$

$(x - \sqrt{3})\left(x \frac{-1}{\sqrt{3} \frac{-1}{\sqrt{3}}}\right)$

{from Equation (2) & (3)}

$\frac{1}{\sqrt{3}} \times \left(x \frac{-\sqrt{3}}{2}\right)$

$\frac{1}{\sqrt{3}} x - \frac{1}{2}$

$\frac{1}{\sqrt{3}}\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) \frac{-1}{2}$

$1 + \frac{1}{3} - \frac{1}{2}$

$= \frac{5}{6}$

2020 - NOVEMBER

[89] if a : b = 3 : 7, then 3a + 2b : 4a + 5b = ?

(a) 23 : 47
(b) 27 : 43
(c) 24 : 51
(d) 29 : 53 (1 mark)

Answer:

(a) If a : b = 3 : 7

let a = 3k, b = 7k

$$\frac{3a+2b}{4a+5b} = \frac{3\times 3k+2\times 7k}{4\times 3k+5\times 7k} = \frac{9k+14k}{12k+35k}$$

$$= \frac{23k}{47k}$$

= 23 : 47

[90] If $\log_a \sqrt{3} = 1/6$, find the value of 'a':

(a) 9

(b) 81

(c) 27

(d) 3 (1 mark)

Answer:

(c) If $\log_a \sqrt{3} = \frac{1}{6}$

$\sqrt{3} = a^{1/6}$

$a^{1/6} = \sqrt{3}$

$a^{1/6} = 3^{1/2}$

$a = (3^{1/2})^6$

$\boxed{a = 3^3}$

$a = 27$

[91] log 9 + log 5 is expressed as:

(a) log 4

(b) log 9/5

(c) log 5/9

(d) log 45 (1 mark)

Answer:

(d) log 9 + log 5 = log (9 × 5)

= log 45

∵ [log m + log n = log (m × n)]

[92] if a : b = 9 : 4, then $\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}$=?

(a) 3/2
(b) 2/3
(c) 6/13
(d) 13/6 (1 mark)

Answer:

(d) If a : b = 9 : 4
let a = 9k, b = 4k

$$\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}=\sqrt{\frac{9k}{4k}}+\sqrt{\frac{4k}{9k}}$$

$$=\frac{3}{2}+\frac{2}{3}=\frac{9+4}{6}=\frac{13}{6}$$

[93] The ratio of number of boys and the number of girls in a school is found to be 15 : 32. How many boys and equal number of girls should be added to bring the ratio to 2/3?

(a) 19
(b) 20
(c) 23
(d) 27 (1 mark)

Answer:

(a) On calculator, we find that 2/3 = 0.67
Let the number added to each term of the ratio 15 : 32 be x.
Now, try the options.
Option (a) → 19

$$\frac{15+19}{32+19}=0.67$$

Therefore, option (a) is the answer.

[94] Find the value of a from the following:

$$(\sqrt{9})^{-5}\times(\sqrt{3})^{-7}-(\sqrt{3})^{-a}$$

(a) 11
(b) 13
(c) 15
(d) 17 (1 mark)

Answer:

(d) $(\sqrt{9})^{-5} \times (\sqrt{3})^{-7} = (\sqrt{3})^{-a}$

$3^{-5} \times \left(3^{\frac{1}{2}}\right)^{-7} = \left(3^{\frac{1}{2}}\right)^{-a}$

$3^{-5} \times 3^{-\frac{7}{2}} = 3^{-\frac{a}{2}}$

$3^{-5-\frac{7}{2}} = 3^{-\frac{a}{2}}$

$3^{\frac{-10-7}{2}} = 3^{-\frac{a}{2}}$

$3^{-\frac{17}{2}} = 3^{-\frac{a}{2}}$

$-\frac{17}{2} = -\frac{a}{2}$

$a = 17$

2021 - JANUARY

[95] Find the value of $\frac{3t^{-1}}{t^{-1/3}}$

(a) $\frac{3}{t^{2/3}}$

(b) $\frac{3}{t^{3/2}}$

(c) $\frac{3}{t^{1/3}}$

(d) $\frac{3}{t^{2}}$ (1 mark)

Answer:

(a) $\frac{3t^{-1}}{t^{-1/3}} = \frac{3}{t^{1-\frac{1}{3}}} = \frac{3}{t^{\frac{2}{3}}}$

[96] If $\log_a (ab) = x$, then $\log_b (ab)$ is

(a) 1/x

(b) $\frac{x}{1+x}$

(c) $\frac{x}{x-1}$

(d) None of these (1 mark)

Answer:

(c) $\log_a (ab) = x$

$\log_a a + \log_a b = x$ [As log m + log n = log mn]

$1 + \log_a b = x$

$\log_a b = x-1$ ... Eq. (1)

We know that $\log_a b \times \log_b a = 1$

Putting the value of $\log_a b$ from eq. (1), we get:

$(x - 1) \times \log_b a = 1$

$\log_b a = \frac{1}{x-1}$

$\log_a (ab) = \frac{\log_b (ab)}{\log_b a}$ [As per Base Change Formula]

$\log_b (ab) = \log_a (ab) \times \log_b a$

$\log_b (ab) = x \times \left(\frac{1}{x-1}\right)$ $\left[\text{As } \log_b (ab) = x \text{ and } \log_b a = \frac{1}{x-1}\right]$

$\log_a (ab) = \frac{x}{x-1}$

[97] In a certain business A and B received profit in a certain ratio B and C received profits in the same ratio. If A gets ₹ 1600 and C gets ₹ 2500 then how much does B get?

(a) ₹ 2,000

(b) ₹ 2,500

(c) ₹ 1,000

(d) ₹ 1,500 (1 mark)

Answer:

(a) $\frac{A}{B} = \frac{B}{C}$

$B^2 = A \times C$

$B = \sqrt{A \times C} = \sqrt{1,600 \times 2,500} = 2,000$

[98] The ratio of two quantities is 15 : 17. If the consequent of its inverse ratio is 15, then the antecedent is;

(a) 15

(b) $\sqrt{15}$

(c) 17

(d) 14 (1 mark)

Answer:

(c) Inverse Ratio = $\frac{17}{15}$

Therefore, antecedent = 17

[99] The salaries of A, B and C are in the ratio 2 : 3 : 5. If increments of 15%, 10% and 20% are allowed respectively to their salary, then what will be the new ratio of their salaries?

(a) 3 : 3 : 10

(b) 10 : 11 : 20

(c) 23 : 33 : 60

(d) Cannot be determined (1 mark)

Answer:

(c) Since the ratio of the salaries of A, B and C is 2 : 3 : 5, let the salaries be 200, 300, and 500 respectively.

A's new salary = 200 + (15% of 200) = 230

B's new salary = 300 + (10% of 300) = 330

C's new salary = 500 + (20% of 500) = 600

Therefore, clearly, the new ratio is 23 : 33 : 60.

2021 - JULY

[100] The salaries of A, B, and C are in the ratio 2 : 3 : 5. If increments of 15%, 10% and 20% are allowed respectively to their salary, then what will be the new ratio of their salaries?

(a) 23 : 33 : 60
(b) 33 : 23 : 60
(c) 23 : 60 : 33
(d) 33 : 60 : 23 (1 mark)

Answer:

(a) Since the ratio of the salaries of A, B and C is 2 : 3 : 5, let the salaries be 200, 300, and 500 respectively.
A's new salary = 200 + (15% of 200) = 230
B's new salary = 300 + (10% of 300) = 330
C's new salary = 500 + (20% of 500) = 600
Therefore, clearly, the new ratio is 23 : 33 : 60.

[101] If A : B = 5 : 3, B : C = 6 : 7 and C : D = 14 : 9 then the value of A : B : C : D is:

(a) 20 : 14 : 12 : 9
(b) 20 : 9 : 12 : 14
(c) 20 : 9 : 14 : 12
(d) 20 : 12 : 14 : 9 (1 mark)

Answer:

(d) We have $\frac{A}{B} = \frac{5}{3}$ and $\frac{B}{C} = \frac{6}{7}$.

To make the Bs same, let's multiply $\frac{A}{B} = \frac{5}{3}$ with $\frac{2}{2}$

Now, $\frac{A}{B} = \frac{5}{3} \times \frac{2}{2} = \frac{10}{6}$ and $\frac{B}{C} = \frac{6}{7}$.

Also, we have $\frac{C}{D} = \frac{14}{9}$.

To make the Cs same, let's multiply $\frac{B}{C} = \frac{6}{7}$ with $\frac{2}{2}$.

Therefore, $\frac{B}{C} = \frac{6}{7} \times \frac{2}{2} = \frac{12}{14}$.

Now, we have $\frac{A}{B} = \frac{10}{6}$; $\frac{B}{C} = \frac{12}{14}$; $\frac{C}{D} = \frac{14}{9}$.

Again, to make the Bs same, let's multiply $\frac{A}{B} = \frac{10}{6}$ with $\frac{2}{2}$.

Therefore, $\frac{A}{B} = \frac{10}{6} \times \frac{2}{2} = \frac{20}{12}$.

So, now we have $\frac{A}{B} = \frac{20}{12}$; $\frac{B}{C} = \frac{12}{14}$; $\frac{C}{D} = \frac{14}{9}$.

Therefore, A : B : C : D = 20 : 12 : 14 : 9

[102] A vessel contained a solution of acid and water in which water was 64%. Four litres of the solution were taken out of the vessel and the same quantity of water was added. If the resulting solution contains 30% acid, the quantity (in litres) of the solution, in the beginning in the vessel, was

(a) 12
(b) 36
(c) 24
(d) 2 (1 mark)

Answer:

(c) Let the initial total volume be V.
Water = 0.64V; Acid = 0.36V
Now, 4 litres were taken out.
Remaining Water = 0.64V - (0.64 × 4) = 0.64V - 2.56
Remaining Acid = 0.36V - (0.36 × 4) = 0.36V - 1.44
To the above, 4 litres of water was added. Therefore, the total volume of the vessel would be V - 4litres + 4 litres = V.
Now, it is given that this resulting solution contains 30% of acid.

Therefore, $\frac{0.36V - 1.44}{V} = 0.30$

$\Rightarrow$ 0.36V - 1.44 = 0.30V

$\Rightarrow$ 0.36V - 0.30V = 1.44

$\Rightarrow$ 0.06V = 1.44

$\Rightarrow$ $V = \frac{1.44}{0.06} = 24$

[103] If xy + yz + zx = -1 then the value of $\left(\frac{x+y}{1+xy}+\frac{z+y}{1+zy}+\frac{x+z}{1+zx}\right)$ is:

(a) xyz

(b) $\frac{-1}{yz}$

(c) $\frac{1}{xyz}$

(d) $\frac{1}{x+y+z}$ (1 mark)

Answer:

(c) Given xy + yz + zx = -1

This means 1 + xy = -yz - zx ... Eq. (1)

1 + yz = -xy - zx ... Eq. (2)

1 + zx = -xy - yz ... Eq. (3)

$$\frac{x+y}{1+xy}+\frac{z+y}{1+zy}+\frac{x+z}{1+zx}$$

Substituting the values of 1 + xy, 1 + zy, and 1 + zx above from Eqs. (1), (2), and (3), we get:

$$\frac{x+y}{-yz-zx}+\frac{z+y}{-xy-zx}+\frac{x+z}{-xy-yz}$$

$$\Rightarrow \frac{x+y}{-z(y+x)}+\frac{z+y}{-x(y+z)}+\frac{x+z}{-y(x+z)}$$

$$\Rightarrow \frac{-1}{z}+\frac{-1}{x}+\frac{-1}{y}$$

$$\Rightarrow -\left(\frac{1}{z}+\frac{1}{x}+\frac{1}{y}\right)$$

$$\Rightarrow -\left(\frac{xy+yz+zx}{xyz}\right)$$

$$\Rightarrow -\left(\frac{-1}{xyz}\right)$$

$$\Rightarrow \frac{1}{xyz}$$

[104] If $\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$ then the value of x is

(a) 64
(b) 4
(c) 16
(d) 2 (1 mark)

Answer:

(c) $\log_4 x + \log_{16} x + \log_{64} x + \log_{256} x = \frac{25}{6}$

$\Rightarrow \quad \log_{2^2} x + \log_{2^4} x + \log_{2^6} x + \log_{2^8} x = \frac{25}{6}$

$\Rightarrow \quad \frac{1}{2}\log_2 x + \frac{1}{4}\log_2 x + \frac{1}{6}\log_2 x + \frac{1}{8}\log_2 x = \frac{25}{6}$

$\Rightarrow \quad \log_2 x\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}\right) = \frac{25}{6}$

$\Rightarrow \quad \log_2 x\left(\frac{12+6+4+3}{24}\right) = \frac{25}{6}$

$\Rightarrow \quad \log_2 x\left(\frac{25}{24}\right) = \frac{25}{6}$

$\Rightarrow \quad \log_2 x = \frac{25}{6} \times \frac{24}{25}$

$\Rightarrow \quad \log_2 x = 4$

$\Rightarrow \quad x = 2^4 = 16$

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[105] Let a= $(\sqrt{5}+\sqrt{3})/(\sqrt{5}-\sqrt{3})$ and b=$(\sqrt{5}-\sqrt{3})/(\sqrt{5}+(\sqrt{3})$. What is the value of $a^2 + b^2$?

(a) 64
(b) 62
(c) 60
(d) 254 (1 mark)

Answer:

(b) $a = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{3.9681}{0.5040} = 7.8732$

$b = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{0.5040}{3.9681} = 0.1270$

$a^2 + b^2 = (7.8732)^2 + (0.1270)^2 = 62$

[106] Incomes of R and S are in the ratio 7 : 9 and their expenditures are in the ratio 4 : 5. Their total expenditure is equal to income of R. What is the ratio of their savings?

(a) 23 : 36
(b) 28 : 41
(c) 31 : 43
(d) 35 : 46 (1 mark)

Answer:

(d) Let the incomes of R and S be in 7x and 9x respectively, and their expenditures be 4y and 5y respectively.

Savings of R = 7x – 4y

Savings of S = 9x – 5y

Also, it is given that their total expenditures is equal to the income of R.

Therefore, 4y + 5y = 7x

$\Rightarrow 9y = 7x$

$\Rightarrow x = \frac{9y}{7}$ .... Eq. (1)

Ratio of their expenditures = $\frac{7x-4y}{9x-5y}$

Putting the value of $x = \frac{9y}{7}$ from Eq. (1)

above:

$$\frac{7\left(\frac{9y}{7}\right)-4y}{9\left(\frac{9y}{7}\right)-5y}$$

$$= \frac{5y}{\frac{81y}{7} - 5y}$$

$$= \frac{5y}{\frac{81y - 35y}{7}}$$

$$= \frac{7 \times 5y}{46y}$$

$$= \frac{35}{46}$$

[107] A bag contains 105 coins containing some 50 paise, and 25 paise coins. The ratio of the number of these coins is 4 : 3 . The total value (in ₹) in the bag is

(a) 43.25
(b) 41.25
(c) 39.25
(d) 35.25 (1 mark)

Answer:

(b) No. of 50 paise coins = $\frac{4}{7} \times 105 = 60$

No. of 25 paise coins = $\frac{3}{7} \times 105 = 45$

Value of 1 50-paisa coin = ₹ 0.50
Therefore, value of 60 50-paisa coins = 60 × ₹ 0.50 = ₹ 30
Value of 1 25-paisa coin = ₹ 0.25
Therefore, value of 45 25-paisa coins = 45 × ₹ 0.25 = ₹ 11.25
Therefore, total value = ₹ 30 + ₹ 11.25 = ₹ 41.25

[108] If $\log_{10} 3 = x$ and $\log_{10} 4 = y$, then the value of $\log_{10} 120$ can be expressed as

(a) x - y + 1
(b) x + y + 1
(c) x + y - 1
(d) 2x + y - 1 (1 mark)

Answer:

(b) $\log_{10} 120 = \log_{10} (3 \times 4 \times 10)$
$= \log_{10} 3 + \log_{10} 4 + \log_{10} 10$
$= x + y + 1$

[109] Find the value of $\log(x^6)$, if $\log(x) + 2\log(x^2) + 3\log(x^3) = 14$.

(a) 3 (b) 4
(c) 5 (d) 6 (1 mark)

Answer:

(d) $\log (x) + 2 \log (x^2) + 3 \log (x^3) = 14$
$\Rightarrow \log x + (2 \times 2) \log x + (3 \times 3) \log x = 14$
$\Rightarrow \log x + 4 \log x + 9 \log x = 14$
$\Rightarrow 14 \log x = 14$
$\Rightarrow \log x = \frac{14}{14} = 1$
$\text{Log } (x^6) = 6 \log x = 6 \times 1 = 6$

[110] The value of $\frac{6^{n+4} + 3^{n+3} \times 2^{n+3}}{5 \times 6^n + 6^n}$ is:

(a) 232
(b) 242
(c) 252
(d) 262 (1 mark)

Answer:

(c) We can see that none of the options are in terms of n. This means that n is ultimately going to get cancelled out. Therefore, we can take any value and put it in place of n, and we'll get the same answer. For the sake of simplicity, let n = 1.
Now,

$$\frac{6^{n+4} + 3^{n+3} \times 2^{n+3}}{5 \times 6^n + 6^n}$$

$$= \frac{6^{1+4} + 3^{1+3} \times 2^{1+3}}{5 \times 6^1 + 6^1}$$

$$= \frac{6^5 + 3^4 \times 2^4}{5 \times 6 + 6}$$

$$= \frac{7,776 + 81 \times 16}{30 + 6}$$

$$= \frac{7,776 + 1,296}{36}$$

$$= \frac{9,072}{36}$$

$$= 252$$

[111] In a department, the number of males and females are in the ratio 3 : 2. If 2 males and 5 females join the department, then the ratio becomes 1 : 1. Initially, the number of females in the department is

(a) 9
(b) 6
(c) 3
(d) 8 (1 mark)

Answer:

(b) Let the initial number of males and females be 3x and 2x respectively.

As per the question, $\frac{3x + 2}{2x + 5} = \frac{1}{1}$

$\Rightarrow 3x + 2 = 2x + 5$

$\Rightarrow 3x - 2x = 5 - 2$

$\Rightarrow x = 3$

Therefore, initial number of females = $2 \times 3 = 6$

[112] If, $\left(\frac{3a}{2b}\right)^{2x-4} = \left(\frac{2b}{3a}\right)^{2x-4}$, for some a and b, then the value of x is

(a) 8
(b) 6
(c) 4
(d) 2 (1 mark)

Answer:

(d) Looking at the options, you'll find that if x is 2, then the power of the LHS as well as RHS will become 0. Therefore, LHS and RHS both will be 1, and hence, be equal.

[113] The value of $\left(1 - \sqrt[3]{0.027}\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)^2\right)$ is:

(a) 11/16 (b) 13/16
(c) 15/16 (d) 1 (1 mark)

Answer:

(c) $\left(1 - \sqrt[3]{0.027}\left(\frac{5}{6}\right)\left(\frac{1}{2}\right)^2\right)$

$= \left(1 - \sqrt[3]{\frac{27}{1000}}\left(\frac{5}{6}\right)\left(\frac{1}{4}\right)\right)$

$= \left(1 - \sqrt[3]{\frac{27}{1000}}\left(\frac{5}{24}\right)\right)$

$= \left(1 - \left(\frac{3}{10}\right)\left(\frac{5}{24}\right)\right)$

$= \left(1 - \left(\frac{1}{2} \times \frac{1}{8}\right)\right)$

$= 1 - \frac{1}{16}$

$= \frac{16 - 1}{16} = \frac{15}{16}$

Alternatively,

On calculator, calculate $\sqrt[3]{0.027}$, or $(0.027)^{\frac{1}{3}}$. Follow the following steps.

First, enter 0.027 on the calculator, then press the square root button 12 times. You'll get 0.99911857266.

Then, from this, subtract 1, i.e., press - 1.

You'll get - 0.00088142734.

Then, multiply this number with the power, i.e., 1/3. Press × 1 ÷ 3 =. You'll get - 0.00029380911.

Then add 1 to it, i.e., press + 1. You'll get 0.99970619089.

Then press the buttons (× =) 12 times. You'll get 0.30010617315.

This is $(0.027)^{\frac{1}{3}}$.

Now, multiply this number with $\left\{\frac{5}{6}\left(\frac{1}{2}\right)^2\right\}$.

You'll get 0.0625221194. Then press M+.
This will save this number in the memory of your calculator.
Then press 1 - MRC =. You'll get 0.9374778806.
This is your final answer.
Now, try the options.
Option (a) → 11/16

11 ÷ 16 = 0.6875 ≠ 0.9375

Option (b) → 13/16

13 ÷ 16 = 0.8125 ≠ 0.9375

Option (c) → 15/16
15 ÷ 16 = 0.9375
Therefore, option (c) is the answer.

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[114] $\log\left(\frac{p^2}{qr}\right) + \log\left(\frac{q^2}{pr}\right) + \log\left(\frac{r^2}{pq}\right)$ is:

(a) pqr (b) 0
(c) 1 (d) None (1 mark)

Answer:

(b) $\log\left(\frac{p^2}{qr}\right) + \log\left(\frac{q^2}{pr}\right) + \log\left(\frac{r^2}{pq}\right)$

$= \log\left(\frac{p^2}{qr} \times \frac{q^2}{pr} \times \frac{r^2}{pq}\right)$

$= \log\left(\frac{p^2q^2r^2}{p^2q^2r^2}\right)$

$= \log 1$

$= 0$

[115 If $\log_{\sqrt{3}} a = 6$, then 'a' will be:

(a) 27 (b) 36
(c) 15 (d) 1 (1 mark)

Answer:

(a) Given $\log_{\sqrt{3}} a = 6$

$\Rightarrow (\sqrt{3})^6 = a$

$\Rightarrow a = (\sqrt{3})^6$

$\Rightarrow a = \left[(3)^{\frac{1}{2}}\right]^6$

$\Rightarrow a = 3^{\frac{1}{2}\times 6}$

$\Rightarrow a = 3^3$

$\Rightarrow a = 27$

[116] A box contains 25 paise coins and 10 paise coins and 5 paise coins in ratios 3:2:1 and total money is ₹40. How many 5 paise coins are there?

(a) 65 (b) 55
(c) 40 (d) 50 (1 mark)

Answer:

(c) Let the number of 25 paise coins be $3x$, the number of 10 paise coins be $2x$ and the number of 5 paise coins be x.

Value of 25 paise coins (in ₹) = ₹ $0.25 \times 3x$ = ₹ $0.75x$

Value of 10 paise coins (in ₹) = ₹ $0.10 \times 2x$ = ₹ $0.2x$

Value of 5 paise coins (in ₹) = ₹ $0.05 \times x$ = ₹ $0.05x$

Total value = ₹ $0.75x$ + ₹ $0.2x$ + ₹ $0.05x$ = ₹ 40

Therefor, $x = 40$

Therefore, number of 5 paise coins = 40.

[117] If x : y = 4 : 6 and z : x = 4 : 16 find Y?

(a) 4 (b) 6
(c) 16 (d) 1 (1 mark)

Answer:

(b) If x:y = 4:6 and z:x = 4:16 find y

$\Rightarrow$ z:x = 1:4

so, y:x = 6:4 and x:z = 4:1
y:x:z = 6:4:1
so, y = 6

[118] If $(\sqrt{3})^{18} = (\sqrt{9})^{x}$, find x?

(a) 18 (b) 9
(c) 8 (d) 19 (1 mark)

Answer:

(b) If $(\sqrt{3})^{18} = (\sqrt{9})^{x}$

$\left(3^{\frac{1}{2}}\right)^{18} = (3)^{x}$

$3^{9} = 3^{x}$

On comparing

$9 = x$

[119] $\log_{\sqrt{2}} 64$ is equal to:

(a) 12 (b) 6
(c) 1 (d) 8 (1 mark)

Answer:

(a) $\log_{\sqrt{2}} 64 = \frac{\log 64}{\log \sqrt{2}} = \frac{\log 2^{6}}{\log (2)^{\frac{1}{2}}} = \frac{6 \log 2}{\frac{1}{2} \log 2} = 6 \times 2 = 12$

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[120] If $\log_{10} 2 = y$ and $\log_{10} 3 = x$, then the value of $\log_{10} 15$ is:

(a) x – y + 1 (b) x + y + 1
(c) x –y – 1 (d) y – x + 1 (1 mark)

Answer:

(a) Here, $\log_{10} 2 = y$ and $\log_{10} 3 = x$

then $\log_{10} 15 = \log_{10} (3 \times 5)$

$= \log_{10} 3 + \log_{10} 5$

$= \log_{10} 3 + \log_{10} (10/2)$

$= \log_{10} 3 + \log_{10} 10 - \log_{10} 2$

$= x + 1 - y$

$= x - y + 1$

[121] $\log_3 {}^4 . \text{Log}_4 {}^5 . \log_5 {}^6 . \text{Log}_6 {}^7 . \log_7 {}^8 . \text{Log}_8 {}^9$ equal to:

(a) 3 (b) 2

(c) 1 (d) 0 (1 mark)

Answer:

(b) Here, $\log_3 {}^4 \cdot \log_4 {}^5 \cdot \log_5 {}^6 \cdot \log_6 {}^7 \cdot \log_7 {}^8 \cdot \log_8 {}^9$

$= \frac{\cancel{\log 4}}{\log 3} \times \frac{\cancel{\log 5}}{\cancel{\log 4}} \times \frac{\cancel{\log 6}}{\cancel{\log 5}} \times \frac{\cancel{\log 7}}{\cancel{\log 6}} \times \frac{\cancel{\log 8}}{\cancel{\log 7}} \times \frac{\log 9}{\cancel{\log 8}}$

$= \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2\log 3}{\log 3} = 2$

[122] A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets ₹ 1,000 more than D, what is B's share?

(a) ₹ 2,000 (b) ₹ 1,500

(c) ₹ 2,500 (d) ₹ 1,000 (1 mark)

Answer:

(a) Given, A : B : C : D = 5 : 2 : 4 : 3

Let, A = 5x, C = 4x

B = 2x, D = 3x

$\because$ C gets ₹ 1,000 more than D

$\therefore$ C = D + 1,000

4x = 3x + 1,000

4x − 3x = 1,000

x = 1,000

Share's of B = 2x

= 2 × 1,000

= ₹ 2,000

[123] By simplifying $(2a^3 b^4)^6 / (4a^3 b)^2 \times (a^2 b^2)$, the answer will be:

(a) $4a^2 b^3$ (b) $4a^6 b^4$

(c) $4a^{10} b^{10}$ (d) $4a^{10} b^{20}$ (1 mark)

Answer:

(d) Here, $\frac{(2a^3 b^4)^6}{(4a^3 b)^2 \times (a^2 b^2)} = \frac{2^6 a^{18} b^{24}}{(4^2 a^6 b^2 \times a^2 b^2)}$

$= \frac{64 a^{18} b^{24}}{16 a^8 b^4}$

$= 4a^{10} \cdot b^{20}$

[124] A group of 400 soldiers posted at border area had a provision for 31 days. After 28 days 280 soldiers from this group were called back. Find the number of days for which the remaining ration will be sufficient?

(a) 3 (b) 6
(c) 8 (d) 10 (1 mark)

Answer:

(d) Here, Total men = 400, No. of days = 31
Total No. of unit of food for 400 men in 31 days
= 400 × 31 = 12400 unit
Total No. of unit of food for 400 men in 28 days
= 400 × 28 = 11200 unit
Rest food = 12400 -11200 = 1200 unit
Remain men after 28 days = 400 - 280 = 120
No. of days for which the remaining food will be sufficient

$$= \frac{\text{Total Rest food}}{\text{No. of Remaining men}}$$

$$= \frac{1200}{120}$$

= 10 days

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[125] If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$ then the value of $\left(\frac{a+b+c}{3}\right)^3$ is equal to:

(a) abc (b) 9abc
(c) 1/abc (d) 1/9abc (1 mark)

Answer:

(a) If $\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c} = 0$

$(a)^{1/3} + (b)^{1/3} + (c)^{1/3} = 0$

let $a^{1/3} = x$, $b^{1/3} = y$, $c^{1/3} = z$

then $x + y + z = 0$

and $a = x^3$, $b = y^3$, $c = z^3$

Now if $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

Now $\left(\frac{a+b+c}{3}\right)^3 = \left(\frac{x^3+y^3+z^3}{3}\right)^3 = \left(\frac{3xyz}{3}\right)^3$

$= (a^{1/3}. b^{1/3}. c^{1/3})^3$

$= (abc)^{1/3 \times 3} = abc$

[126] Given that $\log_{10} x = m + n - 1$ and $\log_{10} y = m - n$, the value of $\log_{10}\left(\frac{100x}{y^2}\right)$ expressed in terms of m and n is:

(a) 1- m + 3n
(b) m -1 + 3n
(c) m + 3n +1
(d) $m^2 - n^2$ (1 mark)

Answer:

(a) Given $\log_{10} x = m + n - 1$ and $\log_{10} y = m - n$

then $\log_{10}\left(\frac{100x}{y^2}\right) = \log_{10} 100x - \log_{10} y^2$

$= \log_{10} 100 + \log_{10} x - 2 \log_{10} y$
$= 2 + \log_{10} x - 2 \log_{10} y$
$= 2 + m + n - 1 - 2 (m - n)$
$= 2 + m + n - 1 - 2m + 2n$
$= 1 - m + 3n$

[127] The Value of $\{\log_6 \{3 \log_{10} 100\}\}$

(a) 1
(b) 2
(c) 10
(d) 100 (1 mark)

Answer:

(a) $[\log_6 \{3\log_{10} 100\}] = \log_6 \{3\log_{10} 10^2\}$
$= \log_6 \{3 \times 2 \log_{10} 10\}$
$= \log_6 \{6 \times 1\}$
$= \log_6 6$
$= 1$

[128] If $x = y^a$, $y = z^b$, $z = x^c$ then the value of abc is

(a) 1

(b) 2

(c) 3

(d) 4 (1 mark)

Answer:

(a) $x = y^a$, $y = z^b$, $z = x^c$ find abc

$x = y^a$, $y = z^b$, $z = x^c$

$\log x = \log y^a$, $\log y = \log z^b$, $\log z = \log x^c$

$\log x = a \log y$, $\log y = b \log z$ $\log z = c \log x$

$a = \frac{\log x}{\log y}$, $b = \frac{\log y}{\log z}$, $c = \frac{\log z}{\log x}$

$abc = \frac{\log x}{\log y} \times \frac{\log y}{\log z} \times \frac{\log z}{\log yx} = 1$

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[129] If $2^x = 4^y = 8^z$ and $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$, then the value of z is:

(a) $\frac{7}{16}$

(b) $\frac{7}{32}$

(c) $\frac{7}{48}$

(d) $\frac{7}{64}$ (1 mark)

Answer:

(c) If $2^x = 4^y = 8^z$

$2^x = 2^{2y} = 2^{3z}$

$\Rightarrow x = 2y = 3z$

$\Rightarrow 2x = 4y = 6z$

$$\text{Now } \frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z} = \frac{24}{7}$$
$$\frac{1}{6z}+\frac{1}{6z}+\frac{1}{6z} = \frac{24}{7}$$
$$\frac{1+1+1}{6z} = \frac{24}{7}$$
$$\frac{3}{6z} = \frac{24}{7}$$
$$6z \times 24 = 3 \times 7$$
$$z = \frac{3\times 7}{6\times 24} = \frac{7}{48}$$

[130] If $\frac{9^n\times 3^5\times (27)^5}{3\times (81)^4} = 27$, then the value of n is:

(a) 2
(b) 0
(c) 3
(d) 4 (1 mark)

Answer:

(b) If $\frac{9^n\times 3^5\times (27)^5}{3\times (81)^4} = 27$

$$\frac{3^{2n}\times 3^5\times (3^3)^5}{3\times (3^4)^4} = 3^3$$
$$\frac{3^{2n}\times 3^5\times 3^{15}}{3^1\times 3^{16}} = 3^3$$
$$3^{2n+5+15-1-16} = 3^3$$
$$3^{2n+3} = 3^3$$

on comparing

$2n + 3 = 3$

$2n = 0 \Rightarrow n = 0$

[131] Given $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$. Then find the value of $\frac{1}{x^2}+\frac{1}{y^2}$:

(a) 63 (b) 61
(c) 62 (d) 60 (1 mark)

Answer:

(c) Given,

$x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$x + y = \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})} + \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})}$

$= \frac{(\sqrt{5}+\sqrt{3})^2+(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5}-\sqrt{3})+(\sqrt{5}+\sqrt{3})}$

$= \frac{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}\sqrt{3}+\sqrt{5^2}+(\sqrt{3^2})-2\sqrt{5}\sqrt{3}}{(\sqrt{5^2})-(\sqrt{3^2})}$

$= \frac{5+3+5+3}{5-3} = \frac{16}{2} = 8$

$x - y = \frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})} \times \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})}$

$xy = 1$

$\frac{1}{x^2}+\frac{1}{y^2} = \frac{y^2+x^2}{x^2y^2}$

$= \frac{(x+y)^2-2xy}{(xy)^2}$

$= \frac{(8)^2-2\times 1}{(1)^2}$

$= 64 - 2$

$= 62$

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[132] If $\log_a b = 3$ and $\log_a c = 2$, then $\log_a c$ is:

(a) 5

(b) 6

(c) 9
(d) 1 (1 mark)

Answer:

(b) Given, $\log_a b = 3$ and $\log_b c = 2$

$$\log_a b \times \log_b c = 3 \times 1$$

$$\frac{\cancel{\log b}}{\log a} \times \frac{\log c}{\cancel{\log b}} = 6$$

$$\frac{\log c}{\log a} = 6$$

$$\log_a c = 6$$

[133] If $2^8 = 4^7 = 8^4$ and $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$, then the value of z is :

(a) 7/16
(b) 7/32
(c) 7/48
(d) 7/64 (1 mark)

Answer:

(c) Given, if $2^x = 4^y = 8^z$

$$2^x = 2^{2y} = 2^{3z}$$

$$\Rightarrow \quad x = 2y = 3z$$

or $2x = 4y = 6z$ _____ (1)

Given,

$$\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$$

$$\frac{1}{6z} + \frac{1}{6z} + \frac{1}{6z} = \frac{24}{7}$$

$$\frac{3}{6z} = \frac{24}{7}$$

$$144z = 21$$

$$z = \frac{21}{144} = \frac{7}{48}$$

[134] If the four numbers 1/4, 1/6, 1/10 and 1/x are proportional, then what is the value of x?

(a) 14
(b) 15

(c) 10

(d) 1/12 (1 mark)

Answer:

(b) If, $\frac{1}{4}, \frac{1}{6}, \frac{1}{10}, \frac{1}{x}$ are in proportion

$$\frac{1}{4} \times \frac{1}{x} = \frac{1}{6} \times \frac{1}{10} \quad \boxed{ad = bc}$$

$$\frac{1}{4x} = 60$$

$$4x = 60$$

$$\boxed{x = 15}$$